Announcements

1) Problems to turn in. #'s 5, 11

2) Question added to lab

Finishing Up Problem From Last Thursday:

$$y = \sqrt{64 + (\sqrt{161} - x)^2}$$

$$3x^2 - 6\sqrt{161}\,x + 419 = 0$$

Solving for x: quadratic formula:

$$x = \frac{6\sqrt{161} \pm \sqrt{5796 - 5028}}{6}$$

$$x = \frac{6\sqrt{161} \pm \sqrt{768}}{6}$$

$$= \frac{\sqrt{5796} \pm \sqrt{768}}{6}$$

Is it true that both values are smaller than $\sqrt{161}$ = length of base of triangle?

$$x = \sqrt{161 \pm \frac{\sqrt{768}}{6}}$$

But $\sqrt{161 + \frac{\sqrt{768}}{6}} > \sqrt{161}$,

so throw away, answer is

$$\boxed{x = \sqrt{161 - \frac{\sqrt{768}}{6}}}$$

$$\boxed{y = \sqrt{64 + \left(\frac{\sqrt{768}}{6}\right)^2}}$$

To check that this is a minimum, use the second derivative test.

Calculate $\frac{d^2T}{dx^2}$,

plug in the value of x, if < 0, max

if > 0, min.

General Strategy:

1) Draw a picture, label it.

2) Identify the quantity to optimize. Take derivative w.r.t either this or a closely related variable

3) Substitute to obtain a function of 1 variable.

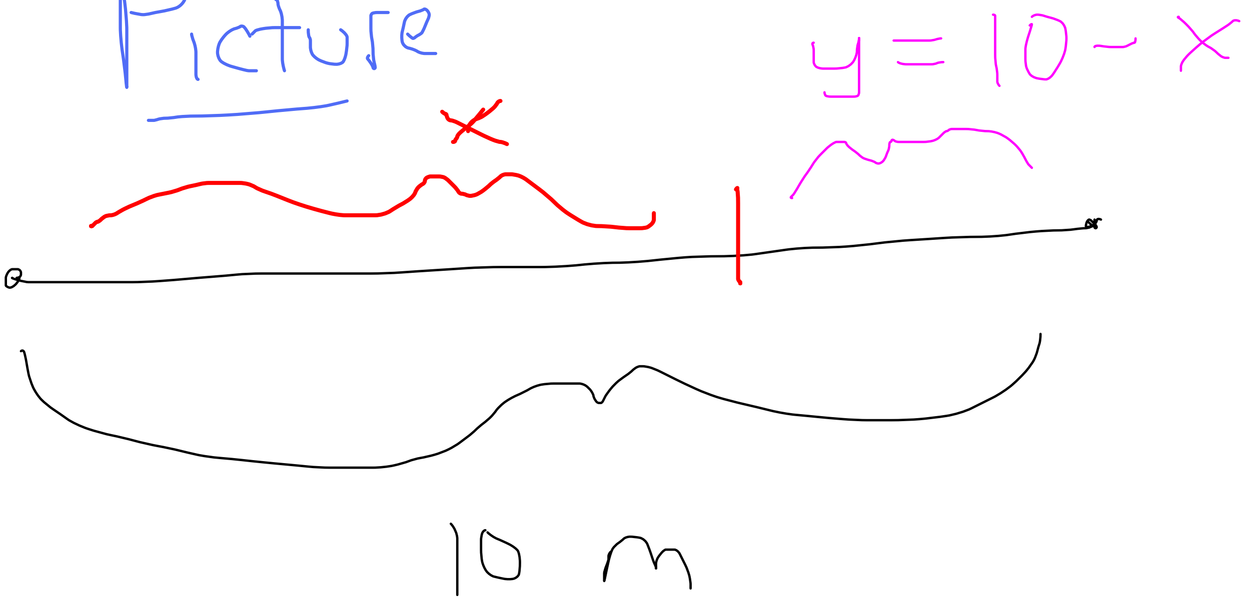
4) Take the derivative, find all critical points (usually just set equal to zero)

5) Find max or min (2^{nd} derivative or 1^{st} derivative tests, endpoints)

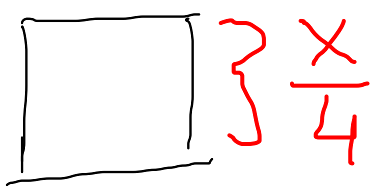
Note If you only have one critical point, it is almost assuredly what you're looking for.

Example 1: You're given a 10 m wire that you want to cut into 2 pieces. If the pieces are to be formed into a square and an equilateral triangle, where do you cut to maximize the combined area of the shapes?

Picture

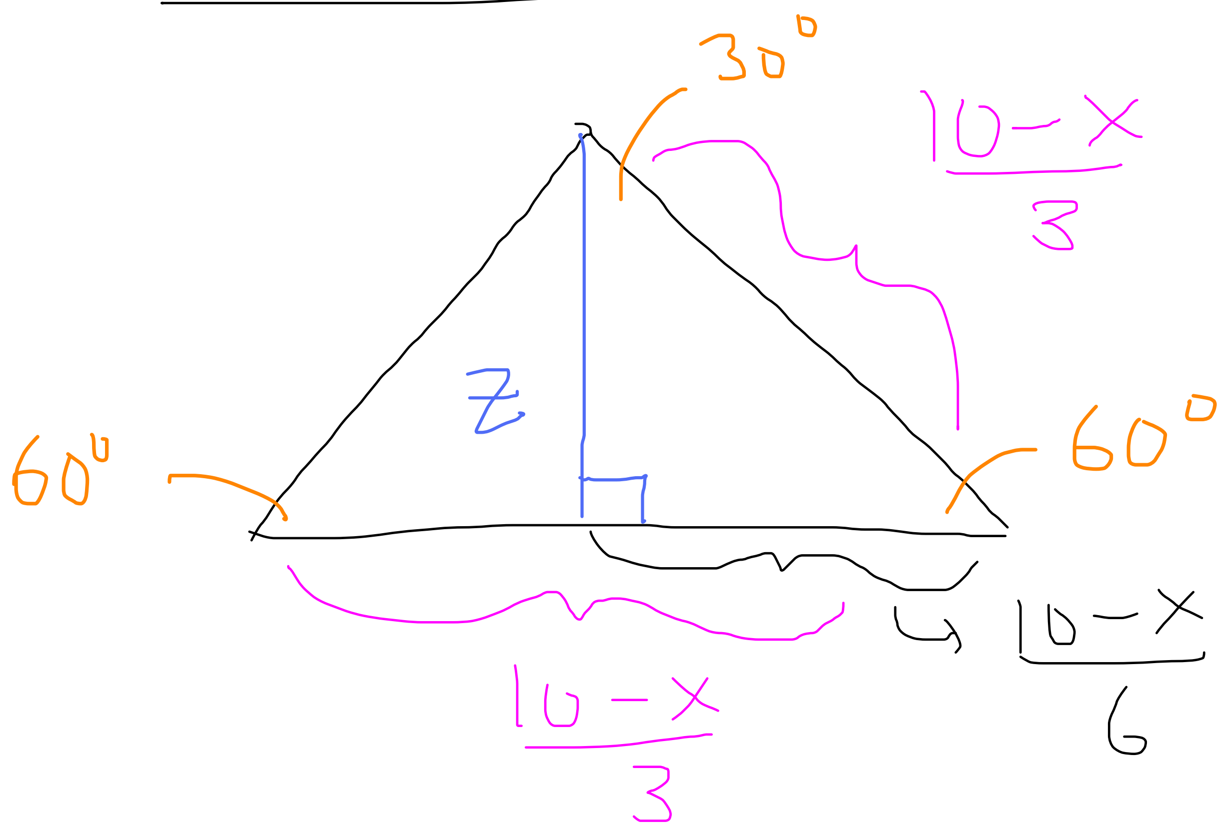


Bend piece of length x into a square:



$$\text{Area} = \frac{x^2}{16} = \left(\frac{x}{4}\right)^2$$

Equilateral Triangle:



$$z^2 + \left(\frac{10-x}{6}\right)^2 = \left(\frac{10-x}{3}\right)^2$$

$$z^2 + \frac{x^2-20x+100}{36} = \frac{x^2-20x+100}{9}$$

Multiply both sides by 36

$$36z^2 + x^2 - 20x + 100 = 4x^2 - 80x + 400$$

$$36z^2 = 3x^2 - 60x + 300$$

Divide both sides by 3

$$12z^2 = x^2 - 20x + 100$$

$$= (x-10)^2$$

So taking square roots,

$$z = \frac{-(x-10)}{\sqrt{12}} = \frac{10-x}{\sqrt{12}}$$

Area of triangle

$$= \frac{1}{2}\left(\frac{10-x}{3}\right)\left(\frac{10-x}{\sqrt{12}}\right)$$

$$= \frac{1}{6\sqrt{12}}(10-x)^2$$

Total area

(Area of triangle) + (Area of square)

$$= \frac{(10-x)^2}{6\sqrt{12}} + \frac{x^2}{16}$$

Differentiate, set to zero.

$$A'(x) = \frac{-2(10-x)}{6\sqrt{12}} + \frac{2x}{16}$$

$$= \frac{x-10}{3\sqrt{12}} + \frac{x}{8}$$

$$= 0$$

Multiply by $3\sqrt{12} \cdot 8$

$$8(x-10) + 3\sqrt{12}\,x = 0$$

so $8x + 3\sqrt{12}\,x = 80$

$$x = \frac{80}{8+3\sqrt{12}}$$

To check whether this is a maximum, calculate $A''(x)$

$$= \frac{1}{8} + \frac{1}{3\sqrt{12}} > 0$$

This means we have a local minimum!

So take the whole wire and stretch into one shape!

Which shape?

Area of square

$$= \left(\frac{10}{4}\right)^2 = \frac{25}{4} = \frac{50}{8}$$

Area of triangle

$$= \frac{100}{6\sqrt{12}} = \frac{50}{3\sqrt{12}} < \frac{50}{8}$$

So bend into a square!

Newton's Method (Section 3.8)

Or "How Your Calculator Works"

You can find the roots of any linear function using basic algebra.

What about

$$x^7 + 2x + 5 = p(x)?$$

$$p(-2) = -128 - 4 + 5 < 0$$

$$p(0) = 5 > 0$$

Intermediate Value Theorem says there is a zero in $(-2, 0)$. How can you approximate the zero?

Pick a point x_1 in $(-2, 0)$.

Look at the point

$(x_1, p(x_1))$.

Make the tangent line to the graph of p at this point.

Line: $p'(x_1)(x - x_1) = y - p(x_1)$

Find the x-intercept

Set $y=0$.

$$p'(x_1)(x-x_1) = -p(x_1)$$

$$x = x_1 - \frac{p(x_1)}{p'(x_1)}$$

Call this point x_2. Repeat procedure to get

$$x_3 = x_2 - \frac{p(x_2)}{p'(x_2)}$$

Example 2:

$$x^7 + 2x + 5 = p(x)$$

choose $x_1 \in (-2, 0)$

$$x_1 = -1$$

$$x_2 = x_1 - \frac{p(x_1)}{p'(x_1)}$$

$$p'(x) = 7x^6 + 2$$

$$x_2 = -1 - \frac{2}{9} = -\frac{11}{9}$$

$$x_3 = -\frac{11}{9} - \frac{p(-11/9)}{p'(-11/9)}$$